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DYNAMIC STABILITY MEASUREMENTS FROM TUNNEL UNSTEADINESS EXCITED RANDOM RESPONSE

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Abstract

Flow unsteadiness in transonic and supersonic tunnels often causes large scatter and when excessive, even impedes dynamic stability measurements by conventional oscillatory techniques. A novel technique utilising the tunnel unsteadiness as primary excitation on flexure-mounted models for dynamic stability measurements in a trisonic blow-down tunnel is presented in this paper. A time series Autoregressive modelling technique is used for deriving a digital spectrum of the unsteadiness excited model response and the system damping is evaluated from the half-power bandwidth of the spectrum. A typical record length used for the spectral analysis is 1.5 seconds. The technique, validated by comparisons with conventional free-oscillation pitch-damping measurements on two models at subsonic and transonic speeds, is well suited for dynamic stability measurements on stable configurations in short duration intermittent facilities.

Nomenclature

$a_1 .. a_n$	Autoregressive parameters corresponding to model order n
D	Structural damping
d	Reference length, base diameter of model
C_m	Pitching-moment coefficient, pitching-moment/ $q_\infty Sd$
C_{m_q}	Pitching-moment coefficient due to pitch velocity $\partial(C_m)/\partial(qd/V_\infty)$
$C_{m_{\dot\alpha}}$	Pitching-moment coefficient due to rate of change of angle of attack, $\partial(C_m)/\partial(\dot\alpha d/V_\infty)$
I	Structural inertia
M_∞	Free-stream Mach Number
N	Observation Vector Length
n	Model order of the Autoregressive description
q	Pitching Velocity
q_∞	Free-stream dynamic pressure
R_{Nd}	Reynold number based on d
R_e	Autocorrelation function
r	Residuals
S	Reference area, $\pi d^2/4$
Δt	Sampling period
V_∞	Free-stream velocity
V_n	Loss function
α	Angle of attack
ω	Model natural frequency, radians/sec
$\Delta\omega$	Half-power bandwidth .
ζ	Damping ratio
$\hat\lambda_{AR}$	Estimated Autoregressive parameter vector
τ	Time delay
Φ_{AR}	Autoregressive spectrum

I. Introduction

Wind tunnel measurement of dynamic stability derivatives are usually based on conventional free-oscillation and forced-oscillation methods. These techniques employ spring-mounted models usually constrained to a single or sometimes two and three degree-of-freedom motion and the derivaties are obtained by analysis of the model response to an initial step displacement or a steady sinusoidal forcing input. The unavoidable flow fluctuations in the tunnel free-stream constitute one of the chief sources of errors adversely affecting the measurement accuracy of these methods. The tunnel unsteadiness which is random in nature acts as an additional and unwanted source of excitation on the model and the response due to this is superposed on the deterministic response of the model (decaying or constant amplitude oscillations) resulting in low signal to noise ratio and consequently large scatter in measurements. The usuai procedure to reduce inaccuracies on this account is to conduct lonqer duration tests with several damping cycles in free-oscillation method

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or long data recording in forced-oscillation method and the stability derivatives are evaluated by an averaging process.. Such procedures in addition to requiring long test durations may not be helpful in some cases when the unsteadiness is high and the signal is largely submerged in noise. Levels of unsteadiness beyond which dynamic stability and other dynamic measurements may not be possible are sometimes specified[1, 2].

Concept of using environmental random disturbance as a test input is currently gaining ground in many branches of engineering[3]. This technique which dispenses with expensive external excitation systems, is becoming increasingly popular in flight flutter testing where the atmospheric turbulence is utilised as the primary excitation (Ref. 4 and 5 are examples). Application of this technique for wind tunnel tests has mostly been for sub-critical damping measurements of flutter models[6,7] and the only attempt to adapt this technique for dynamic stability measurements seems to be that of Ref. 8.

Several techniques are known for analysis of random response of linear systems such as power spectral density, auto-correlation, Random Decrement method etc. Relative merits of these and other technique- are discussed in Ref. 9 and comparative evaluations of some of these methods are also found in Ref. 6, 7 and 10. The Random Dec method developed by H.A. Cole, Jr.[11] can be adapted for on-line measurements or for digital computation based off-line analysis. Requirement of long record lengths Seems to inhibit a wider application of this method to short duration facilities. For example it has been estimated[12] that record lengths of 30 to 150 seconds are required for sub-critical damping measurements on typical wind tunnel flutter models. Drane and Hutton[10] reach a similar conclusion, "the time needed to gather sufficient data for analysis would preclude use of the method in all but the longest running intermittent tunnels". Spectral and correlation methods depend on time averaging analog electronic circuitry and often do not possess adequately low bandwidth resolution. Use of digital techniques for spectrum evaluation of random signal is much more efficient because of availability of FFT and allied techniques. These techniques can be adapted for short record lengths and hence are best suited for short duration intermittent facilities, such as the High Reynolds number transonic wind tunnels that are being or proposed to be built at several places. These facilities have typically about 10 seconds test duration and though efforts are underway to achieve reduced flow unsteadiness, availability of measurement techniques suitable for 'rough' flow typical of ventilated wall test sections would be advantageous.

This paper presents a description of some experimental investisations for dynamic stability measurements using the flow unsteadiness as the primary excitation on flexure-mounted models in a trisonic blow down tunnel. The mathematical modelling technique used for evaluating the system resonant frequency and damping from the model random response is discussed in detail. The technique is validated by comparisons with conventional free-oscillation measurements at subsonic and transonic speeds.

II. Analytical Background

The method is schematically illustrated in Fig. 1. The test model with its flexure spring support system is assumed to represent a stable linear dynamical system of unknown degrees of freedom excited solely by the tunnel unsteadiness (though damping evaluation for the present tests was based on a single degree of freedom assumption, the mathematical modelling technique presented here holds good for the general case of a multi-degree of freedom system). Further, the excitation caused by the tunnel unsteadiness is assumed to be a weakly stationary "white noise" process. A well-known property of asymptotically stable linear systems excited by stationary white noise process is that the output of the system, in this case the model response, is a Stochastic process of rational spectrum which can be described by an Autoregressive model[13,14]. The model response is used for a mathematical model of unknown degrees of freedom whose output correctly matches the recorded response in a least square sense. Time series analysis concepts are used to fit an Autoregressive model whose order is determined by analysing the residuals of the mathematical fit. The mathematical model is used to evaluate the Autoregressive spectrum from which the system resonant frequency and half-power bandwidth and hence the system damping are extracted.

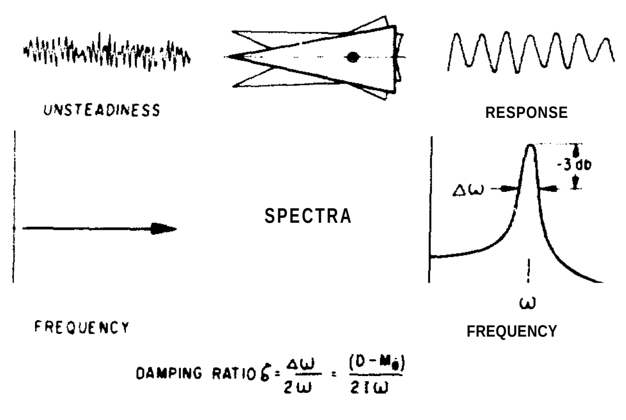


Fig. 1 Schematic Illustration of the Method

Autoregressive (AR) Modelling

The output of the system is taken to be that of an nth order linear dynamical system excited by an inaccessible white noise representing the flow unsteadiness and is assumed to be stationary over the period of observation. This continuous stationary random process $\{y(t)\}$ is observed as a discrete process $\{y(k)\}$, $k = [0,1,2..,N+n]$ Δt where $N \gg n$, using uniformly spaced samples taken at intervals Δt by a zero order sampler. Sampling interval is chosen to be compatible with the spectrum of interest[14]. Then

$$e(k) = A(z^{-1})\, y(k) \tag{1}$$

represents an nth order Autoregressive process, where

z^{-1} : backshift operator representing sampling time

$A(z^{-1})$: the mathematical model embracing the system parameters

$= 1 + a_1 z^{-1} + a_2 z^{-2} + \dots a_n z^{-n}$

$e(k)$: independently and identically distributed inaccessible white noise representing the environment and possessing zero mean and variance of σ^2

Equation (1) can be rewritten in the prediction form as

$$y(k) = e(k) - \left[a_1 y(k-1) + a_2 y(k-2) + \dots a_n y(k-n)\right] \tag{2}$$

Significance of the Autoregressive model is that the output of the model is dominantly a function of its previous history over the period $n\Delta t$, with an additive but inaccessible white noise component and has a predictor capability.

Equation (2) can be rewritten with $e(k)$ replaced by $r(k)$ to represent residuals in the estimation procedure as

$$y(k) = r(k) - \left[a_1 y(k-1) + a_2 y(k-2) + \dots a_n y(k-n)\right] \tag{3}$$

If N such observations of $y(k)$ are made consecutively, the set of such observations can be expressed in the vector matrix notation as

$$\underline{Y} = B\underline{\lambda} + \underline{R} \tag{4}$$

where

$$\underline{Y} = \left[y(n+1)\ y(n+2)\ \dots\ y(n+N)\right]^T$$

$$\lambda = \left[-a_1\ -a_2\ \dots\ \dots\ \dots\ -a_n\right]^T$$

$$B = \begin{bmatrix} y(n) & y(n-1) & \dots & y(1) \\ y(n+1) & y(n) & \dots & y(2) \\ \cdot & & & \\ \cdot & & & \\ y(n+N-1) & \dots\dots & & y(N) \end{bmatrix}$$

$$\underline{R} = \left[r(n+1)\ r(n+2)\ \dots\ r(n+N)\right]^T$$

the parameter vector $\underline{\lambda}$ can be evaluated by minimising the loss function,

$$V_n = \tfrac{1}{2} \sum_{k=1}^{N} r^2(k) \text{ as}$$

$$\underline{\lambda}_{AR} = (B^TB)^{-1} B^T\underline{Y} + (B^TB)^{-1} B^T\underline{R} \tag{5}$$

With the assumption that residuals represent the inaccessible white noise the second term in the equation (5) tends to zero for a large observation vector length N, consequently $r(k) = e(k)$ and

$$\hat{\underline{\lambda}}_{AR} = (B^TB)^{-1} B^T\underline{Y} \tag{6}$$

Equation (6) is identical to the Autoregressive model fit that can be obtained from Yule-Walker expressions[14].

The fit error which describes the ability of $\hat{\lambda}_{AR}$ to predict the system response can be evaluated by the quantity,

$$\sum_{k=1}^{N} r^2(k) \Big/ \sum_{k=1}^{N} y^2(k) \tag{7}$$

This is a useful figure of merit.

Model Order Determination

Three methods of determining the model order by analysing the residual vector are available in the literature[14,15]. A model order is usually guessed and estimation of $\hat{\lambda}_{AR}$ carried out using equation (6). From this estimate, residuals $r(k)$ can be evaluated using equation (3).

The first technique is to evaluate the autocorrelation function of the residuals for various time delays. If the residuals are uncorrelated this tends to an impulse function with correlation function value

$$R_e^2(\tau)/R_e^2(0) \text{ less than } 1/\sqrt{N}, \text{ for } \tau \neq 0$$

where $R_e^2(\tau)$ is the autocorrelation function of $e(k)$, $k=0,1,\dots N$ for time lag $\tau = k\,\Delta t$.

The second technique is the Repeated Least Squares method wherein the model order statistics $F_{n_1 n_2}$ is evaluated as

$$F_{n_1 n_2} = \frac{V_{n_1} - V_{n_2}}{V_{n_2}} \; \frac{N-2n_2}{2(n_2-n_1)}, \quad N > n_1$$

where n_2 is the new model order and V_{n_1}, V_{n_2} are the loss functions. This is known to possess F-distribution[16]. Insignificant change in $F_{n_1 n_2}$ is sought by progressively increasing the model order.

The third technique due to Akaike[15] consists of determining an information criterion known as Akaike's Information Criteria (AIC) for a scan of the model order. The AIC is given by

$$AIC\ (n_1) = \frac{N+n_1}{N-n_1} \left[Q_0 + a_1 Q_1 + \ldots a_{n_1} Q_{n_1}\right] \qquad (8)$$

where n_1 = test model order and

$$Q_j = \frac{1}{N} \sum_{k=1}^{N-j} y(k+j)\ y(k), \quad j=0,1\ldots n_1.$$

Autoregressive Spectrum and System Damping Evaluation

Having evaluated the model order n, and the associated parameter vector $\hat{\lambda}_{AR}$, the Autoregressive spectrum associated with the observation signal can be evaluated by frequency domain transformation as[14]

$$\Phi_{AR}(\omega) = \frac{\Delta t\, \sigma^2}{1 + \sum_{k=1}^{n} a_k \exp(ik\,\omega\,\Delta t)}$$

where the variance $\sigma^2 = E(e^2)$ and $i = \sqrt{-1}$.

From the Autoregressive spectrum the various modes of the system and the associated natural frequency and bandwidth can be identified.

For the present system, the model spring combination is assumed to be a single degree of freedom system with constant parameters excited by tunnel unsteadiness. The governing differential equation for pitching oscillations of the model, θ is

$$I \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta = P(t) + M$$

where I, D and K are the structural inertia, damping and stiffness respectively. For small displacements the pitching moment M is written as

$$M = M_\theta\, \theta + M_{\dot{\theta}}\, \dot{\theta}$$

where M_θ and $M_{\dot{\theta}}$ are the aerodynamic stiffness and damping derivatives.

When the forcing function P(t) is a white noise process, the bandwidth at 'half-power' point on the response spectrum can be shown to be[17]

$$\Delta\omega = 2\,\omega\zeta$$

where ω is the resonant frequency and

$$\zeta, \text{ the damping ratio } = \frac{D - M_{\dot{\theta}}}{2\, I\omega}$$

The system damping is hence obtained as

$$(D - M_{\dot{\theta}}) = \Delta\omega\, I$$

The structural terms I and D are measured in a conventional wind-off free-oscillation test and the 'half-power' bandwidth is obtained from the Autoregressive spectrum.

III. Experiments

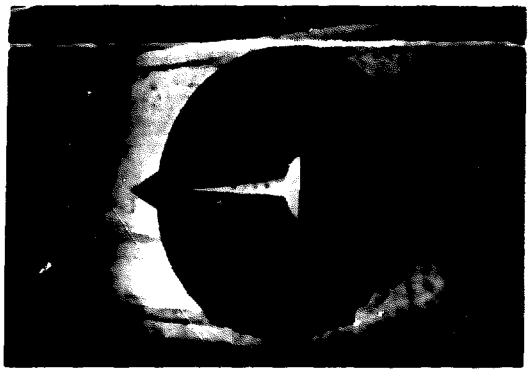
Wind Tunnel

Tests were conductad in the 1-Foot Blowdown Tunnel at NAL. This is an atmospheric discharge tunnel operated from compressed air stored at a maximum pressure of 150 psig. A vertical axis rotating-plug valve controls the tunnel stagnation pressure. A radial splitter flow-spreading system installed in the wide-angle diffuser preceding the settling chamber helps to reduce the flow unsteadiness''. The tunnel is operated at a stagnation pressure of 26 psia at subsonic and transonic speeds. The 12" X 15" transonic test section has 8% slotted roof and floor and solid side walls.

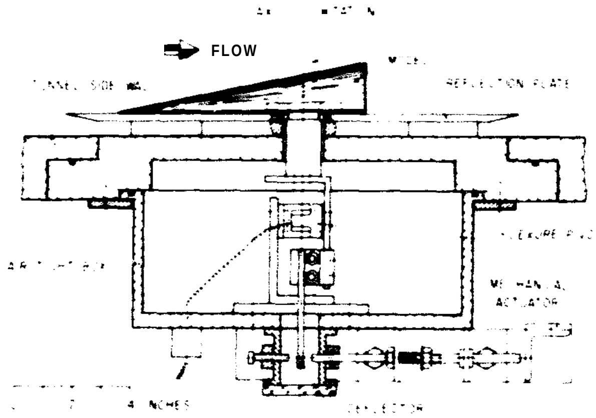
Apparatus

The basic apparatus used for the tests was a wall-mounted oscillation rig shown in Fig. 2. An interchangeable three-strip crossed-flexure pivot anchored on the tunnel sidewall provides the elastic suspension for the model, A reflection plate enables the model to be supported outside the tunnel wall boundary layer. A mechanically actuated spring-loaded device triggers the decaying oscillations from a preset initial amplitude. Relatively stiff flexure pivots were used to reduce the unsteadiness excited model response to values low enough to enable meaningful free-oscillation measurements by conventional log decrement method.

A strain-gauge bridge mounted on the centre strip of the flexure pivot provides the model response signal which is amplified and recorded simultaneously on a FM magnetic tape recorder and an oscillograhic recorder. In order to obtain a qualitative nature of the tunnel unsteadiness, static pressure fluctuation signal



Installation Photograph



Plan View

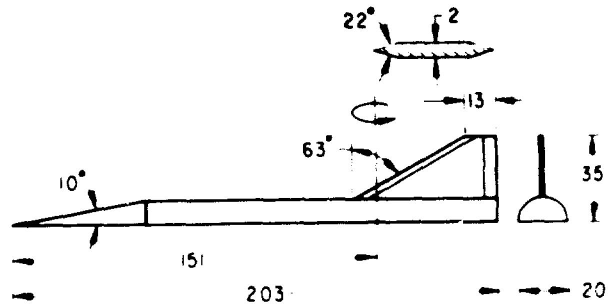
Fig. 2 Oscillation Rig

from a 25 psia ALINCO pressure transducer mounted in a suitable housing on the side-wall was recorded on the tape recorder. The pressure fluctuations were analysed with a B & K Wave Analyzer and Level Recorder. The Autoregressive modelling and the spectrum were evaluated by a program developed on an IBM 360 computer. A typical computation time was 2 minutes.

Models and Wind Tunnel Tests

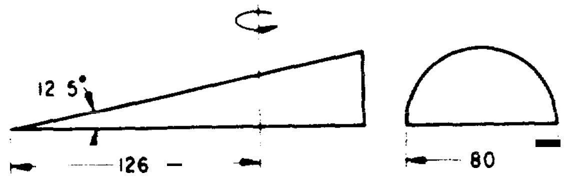
Two half-models (Fig. 3) were used for the tests. The wing-body model had a fineness ratio of 10 and a 63-degree cropped-delta wing with a symmetrical double-wedge section of constant thickness. This was tested with the axis of oscillation at 75% of its length from the nose. The other model was a 25-degree included angle cone with the axis of oscillation at 70% of the length from the apex. The gap between the model plane of symmetry and the reflection plate was 0.75mm for both the models.

In a typical blowdown of about 30 seconds duration the model response to tunnel unsteadiness was recorded for the first 8 seconds after flow establishment



(a) WING-BODY YODEL

ALL DIMENSIONS ARE IN MMS



(b) CONE MODEL

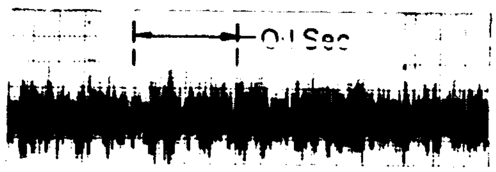
Fig. 3 Details of Models

in the tunnel. In the subsequent 22 seconds test duration conventional decaying oscillation cycles were triggered successively for 10 times. Wind-off free-oscillation records were obtained bef and after each blowdown. The initial amplitude for all free-oscillation tests was 1.5°. The maximum amplitude of the model random response to flow unsteadiness varied with Mach number and was between 0.1 and 0.2 degree.

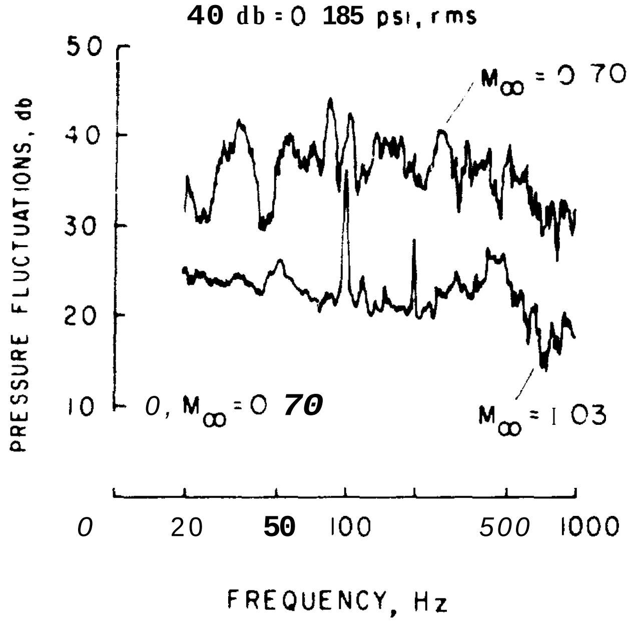
IV Results and Discussions

Typical static pressure fluctuations spectra presented in Fig. 4 show the existence of a wide band frequency in the tunnel flow and though wavy, particularly at M =0.7, they are devoid of many sharp peaks near frequencies of interest (upto about 100Hz). It is pertinent to note that the static pressure fluctuations at best reflect only a qualitative picture of the tunnel unsteadiness and the excitation on the model depends in general on the nature and relative magnitudes of fluctuations in free-stream static and total pressures, velocity and temperature.

The Autoregressive model order was determined using the Repeated least Squares method and the AIC for a scan starting from 5 to 16. Fig. 5 shows the fit error (eqn. (7)) and the AIC (eqn. (8)) as a function of the model order. The observed insignificant change in the fit error and the minima of AIC, which indicates an unambiguous model order, suggest the test choice of model order as 13. The



(a) Record



(b) Spectra

Fig. 4 Typical Pressure Fluctuations

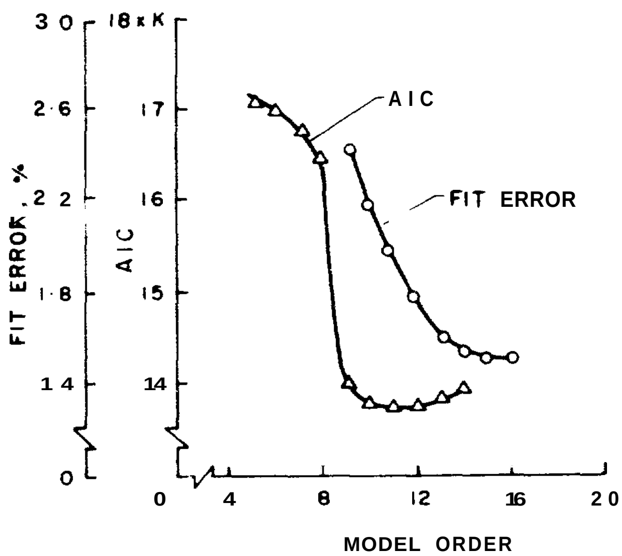
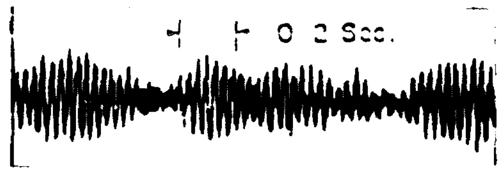
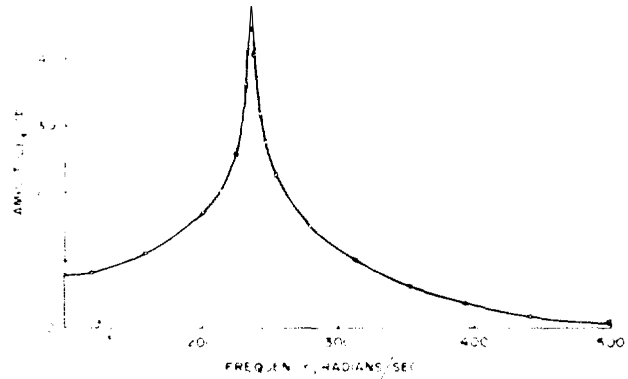


Fig. 5 Model Order Determination



(a) Record



(b) Spectrum

Fig. 6 Typical Random Response of the Model

fit error of the Autoregressive model description was less than 1.5%[19].

A typical record length used for obtaining the Autoregressive spectrum of the model random response was 1.5 seconds. Fig. 6 shows a typical spectrum. The measured bandwidth varied between 0.1 and 0.4Hz corresponding to damping ratios between 0.003 and 0.01. In some cases the response analysis was carried out for three different samples from the same blowdown and the extracted damping ratios agreed within 3%. Results from repeat blowdowns also showed good agreement.

The measured pitch-damping derivatives from the spectral analysis and conventional free-oscillation methods are shown in Fig. 7. The average value of ten damping cycles is presented as one data point for the free-oscillation tests. A typical standard deviation of these values was 20%. Results from the two measurement methods are seen to be in excellent agreement for the two models in the test range of Mach numbers. The good agreement shown despite some discrepancies between the nature of assumed (flat) and the actual (wavy) spectra of tunnel unsteadiness is noteworthy. This indicates that the nature of unsteadiness spectrum a few octaves beyond the model natural frequency may not affect the results. It is believed that this would in general be true for the usual

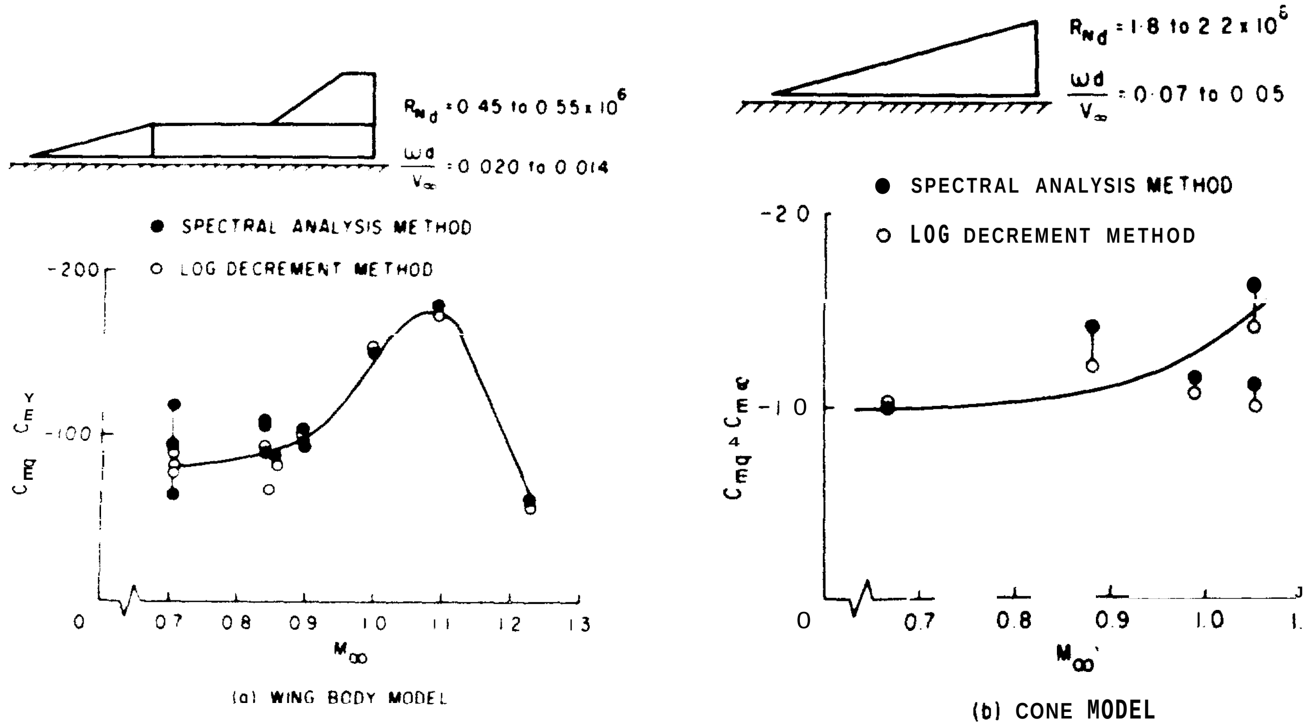


Fig. 7 Comparison of Pitch-Damping Derivatives from Spectral Analysis and Free-Oscillation Methods

high speed tunnel dynamic stability testing systems which have very low damping ratios (or high Q-factors)[20] and consequently the model response to all frequencies but a few octaves around the system resonant frequency is negligible. Unsteadiness spectra in several existing tunnels[21] indicate that the characteristic spectrum at transonic speeds is approximately flat in the region of interest for dynamic stability measurements and advantage can be taken of the existing tunnel unsteadiness for conducting relatively more accurate and less expensive dynamic stability tests, as compared to conventional techniques.

V. Conclusions

The technique of utilising the flow unsteadiness as the primary excitation for dynamic stability measurements in a trisonic wind tunnel has been demonstrated. Autoregressive modelling technique enables the use of short record lengths for deriving a digital spectrum of model random response. The method can in general be used for dynamic stability measurement of stable configurations and would be particularly advantageous when large tunnel unsteadiness and the consequent poor signal to noise ratio renders the conventional free and forced oscillation techniques unsatisfactory or inapplicable. Other advantages of the method are (i) no external excitation equipment is needed (ii) short record lengths (about 1.5 seconds) are adequate for accurate measurements and (iii) bandwidth resolution is not a limitation. The method is well suited for dynamic stability measurements in short duration transonic wind tunnels.

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Nomenclature

$a_1 .. a_n$	Autoregressive parameters corresponding to model order n
D	Structural damping
d	Reference length, base diameter of model
C_m	Pitching-moment coefficient, pitching-moment/ $q_\infty Sd$
C_{m_q}	Pitching-moment coefficient due to pitch velocity $\partial(C_m)/\partial(qd/V_\infty)$
$C_{m_{\dot\alpha}}$	Pitching-moment coefficient due to rate of change of angle of attack, $\partial(C_m)/\partial(\dot\alpha d/V_\infty)$
I	Structural inertia
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N	Observation Vector Length
n	Model order of the Autoregressive description
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ω	Model natural frequency, radians/sec
$\Delta\omega$	Half-power bandwidth .
ζ	Damping ratio
$\hat\lambda_{AR}$	Estimated Autoregressive parameter vector
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Φ_{AR}	Autoregressive spectrum

I. Introduction

Wind tunnel measurement of dynamic stability derivatives are usually based on conventional free-oscillation and forced-oscillation methods. These techniques employ spring-mounted models usually constrained to a single or sometimes two and three degree-of-freedom motion and the derivaties are obtained by analysis of the model response to an initial step displacement or a steady sinusoidal forcing input. The unavoidable flow fluctuations in the tunnel free-stream constitute one of the chief sources of errors adversely affecting the measurement accuracy of these methods. The tunnel unsteadiness which is random in nature acts as an additional and unwanted source of excitation on the model and the response due to this is superposed on the deterministic response of the model (decaying or constant amplitude oscillations) resulting in low signal to noise ratio and consequently large scatter in measurements. The usual procedure to reduce inaccuracies on this account is to conduct longer duration tests with several damping cycles in free-oscillation method

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